Elevation Cable Modeling for Interactive Simulation of Cranes

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Abstract

In this paper, the way to simulate hoisting cables in real time is addressed. We overcome instability in such simulation by considering a two-layered model: a model for the dynamics of a cable passing through a set of pulleys and an oscillation model based on the classical one-dimensional wave equation. The first layer considers the interaction between the cable and pulleys with the elevation equipment, while the second layer simulates cable oscillation. Numerical instability is avoided by suspending the oscillation layer when required. Due to the system properties, this can be carried out in such a way that does not cause significant loss in the system quality. It considers the oscillation of the cable between every pair of pulleys, collision detection and the variation of the cable length very efficiently. Rendering issues are discussed, with remarks on how to prevent aliasing artifacts in the cable. Efficiency is analyzed, including performance tests which show that the model can be run very efficiently. The paper also covers how to integrate the model in a complex multibody simulation with a high degree of interactivity.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism: Animation

Keywords: Physically-based Animation, Flexible Cables, Interactive Simulation

1. Introduction

In the field of computer animation and simulation, models of cables and cable-like structures (ropes, hair strands, surgical thread,...) are necessary in many different applications, such as vehicle design, character animation, cloth simulation and rendering, surgical simulators, and so on. For this reason, different modeling methodologies have been proposed, according to the particular needs and goals of their application. However, when heavy load elevation equipment is simulated, cables are under very high tension and, usually, the cable oscillation is not considered to avoid instability. This leads to a loss of realism in the behavior of the simulated equipment, as cable and pulleys are present in most cranes both for load lift and crane structure movement.

The main concern of this paper is the simulation of a system formed by a cable and several pulleys in high tension situations. A suitable model for this problem should meet the following requirements: modeling of a cable and a set of pulleys; modeling of the cable oscillation; collision detection; variation of cable length, to simulate hoisting; and numerical stability and efficiency, so that it can be introduced in real-time virtual environments. The main goal of the model is not to provide a very accurate simulation of all the aspects involved; instead, our aim is to obtain a feasible animation of cables and to reproduce some important features ignored in previous real-time simulations. For this reason, the elements involved in our methodology are chosen so that simplicity and efficiency, rather than numerical accuracy, is achieved.

This paper is organized as follows. Section 2 overviews the existing methodologies for cable modeling, identifying the main drawbacks for simulating heavy load elevation cables. In Section 3 the cable model is described. In Section 4 some implementation details, including collision detection and rendering are discussed and the stability issues of the wave equation is addressed. In Section 5 the results of several numerical tests are shown. Finally, Section 6 provides some concluding remarks and future work.

2. Previous Work

In this section we provide an overview of previous work on cable and strand modeling, discussing the suitability of the different models for simulating cables under high tension.

2.1. Discrete Models

A common approach to the problem is to consider a cable or strand as a discrete system by means of a chain of particles. The particles are joined by longitudinal and torsional springs to simulate resistance to bending and stretching. This methodology has been used in hair strand modeling for character animation [RCT91, LK01], in thread simulation [HD00, LMGC02, ST08] and in more general deformable objects animation [PW89, Pro95]. This approach, however, leads to stiff systems of differential equations, even in the general case where tension is not as high as in elevation cables.

To overcome the instability resulting from stiff systems, several authors have proposed the use of implicit integration schemes, allowing efficient and stable simulations of cloth [BW98, DSB99] and loose cables [LS01]. These methodologies, however, have not been used for cloth or strands simulation under high tension. In addition, the use of implicit methods introduce further restrictions in the implementation and in the handling of the interactions, such as collisions.

A more general approach is to consider the cable as a chain of rigid bodies, instead of point masses, linked by kinematic constraints. Some authors have considered this approach for loose threads [HMT01, Had06] and even for elevation cables [KH01, SL08]. However, none of these works consider the simulation of long cables going through a series of pulleys. This situation may lead to a high number of bodies per unit length, requiring a lot of computational resources unless reduced coordinates are used [HMT01, Had06].

2.2. Continuous Models

Unlike the discrete approach, strands are also modeled as a continuum object governed by differential equations. Hair strands have been modeled using the cantilever beam equations [AYK92, DMTKT93], but this model is only adequate for one free end strands. Cosserat models [Pai02] represent an almost one-dimensional elastic structure as a parametric curve. A frame is associated to every point in order to obtain potentials to bending, stretching and twisting of loose cables [Pai02, GS07, ST07]. However, these models are usually applied to situations in which bending is more relevant than stretching and no previous results indicate that they are adequate for high-tension cables.

2.3. Fake Dynamics

Another modeling methodology used for general elastically deformable objects is based on the superposition of layered models. These models account for the simulation of different properties of the object by means of simple submodels that are coupled together. This methodology has been used in the animation of complex deformable objects, such as the structure formed by muscles in character animation [CHP89]. Barzel uses this approach [Bar97] to animate ropes and springs by means of what he calls *fake dynamics*. He uses a catenary as a basic shape for a hanging rope, which is modified by sine waves. In the field of hair simulation, this approach has also been applied successfully to obtain complex hairstyles [WS92, KN02, BKCN03]. In these models, however, either there is no dynamics at all or the variation of the parameters or control points is manually determined by the animator.

2.4. Massless Springs and Simulation of Pulleys

More recently, Servin and Lacoursière [SL07] propose a simplified model to simulate an elevation cable. They consider a cable as a massless spring and introduce it as a kinematic constraint on a multibody simulation. They consider the simulation of a set of pulleys and the variation of the cable length, but they neither consider cable oscillation nor collision detection. Furthermore, they implement cable elasticity using the parameters of a particular methodology and numerical scheme for the simulation of the multibody system, which impedes the use of simulation libraries that use a different formulation.

In the field of mechanical engineering, cable modeling is also based on different methodologies. However, models are often numerically expensive and lead to formulations that are not flexible enough for interactive simulation [Dv99, Auf00, GP07]. For this reason, in complex multibody or finite element modeling of cranes, the dynamics of cables is often discarded and massless springs with no oscillation are used. See [ARNM03] for a survey on cable models in crane simulation and control. Models of pulleys have also been developed using the massless spring approach [Auf00, DK00, ZAL04, JC05]. In García-Fernández et al. [GFPCMD07] a layered model is proposed which considers a massless pulley system and the wave equation for cable oscillation. The approach is simple and efficient, but only a simple two-dimensional model with many limitations is presented; the oscillation model only allow a limited movement of the payload while interactivity is not considered. Moreover, no discussion on stability is provided.

2.5. Summary

Although the subject of cable simulation and animation has led to a lot of research, most of the models proposed are not adequate for interactive simulation of elevation cables when heavy loads are lifted. On the one hand, the high mass ratio between the load and the cable leads to stiffness, forcing the use of implicit methods to guarantee stable simulation.

On the other hand, although continuum models lead in some cases to more stable simulations, they are more concerned with the effect of cable bending or twisting, which is not the most relevant feature in our problem. For these reasons, it is difficult to find a model that meets the specifications imposed by elevation devices simulation and that considers cable oscillation, collision detection, variable length cable and the simulation of pulleys.

3. Cable Model

In this section, a new model for the simulation of elevation cables covering the aforementioned needs is proposed. The system to be simulated consists of a succession of pulleys and a cable passing through them. The pulleys and the ends of the cable can be fixed to the world or attached to bodies of the simulation environment. This system works as follows: As a result of external forces (e.g. gravity or contacts) or the movement of pulleys, the cable can oscillate. This oscillation, together with the movement of pulleys, modify the cable stretching, affecting its tension. Tension, in turn, acts on the bodies which have a pulley attached to them, and also modifies the frequency of cable oscillations.

In order to formalize this system the methodology proposed in [GFPCMD07] is used. The dynamics of two physically-based models are coupled: one for the simulation of the cable and pulleys, without considering oscillation, and another for the oscillation of the cable along the segment that joins two adjacent pulleys. The first model can be considered as a basis or skeleton model, while the second model is added to the first one to provide additional detail. The dynamics of both models are coupled to obtain the system described above. Our approach is somehow similar also to the layered dynamics proposed in [CHP89] or [Bar97].

3.1. Cable with Pulleys Model

Initially, the oscillation of the cable is discarded, i.e. it is assumed that the cable goes straight from one pulley to the next. Pulleys are considered frictionless [Auf00] so that the cable moves freely along them and there is the same tension at both sides of a pulley. Our model is a simplification of the finite element model presented in [JC05].

Let us consider a cable of unstretched length equal to L that passes through a set of nodes $\mathbf{P}_i$, $i = 0, \ldots, N$. Nodes $i = 1, \ldots, N-1$ are considered as pulleys, while nodes 0 and N are considered as the points where both ends of the cable are rigidly attached (see Figure 1). Pulleys can either be fixed to the world, be attached to a body and move with it, or have a prescribed motion, $\mathbf{P}_i(t)$. Length L can vary at a rate $\dot{L}$ to simulate the reel/unreel process.

Computation of the Cable Tension

Our stretching model considers a cable as a damped spring. According to the elasticity theory, the stiffness constant of
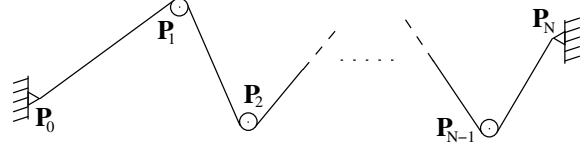

Figure 1: *Scheme of a cable passing through pulleys.*

the spring k is given by the young modulus E of the cable, its cross section area A and its length L, as $k = EA/L$ [FB68]. The damping constant c is obtained experimentally. If we denote the Euclidean distance between nodes i and $i+1$ as $l_i = |\mathbf{P}_{i+1} - \mathbf{P}_i|$ and $\mathbf{d}_i = (\mathbf{P}_{i+1} - \mathbf{P}_i)/l_i$ for $i = 0, \ldots, N-1$, the actual, stretched, length of the cable is $l = \sum l_i$. Its derivative $\dot{l}$ depends on the movement of points $\mathbf{P}_k$. Tension T is computed as

$$T = \begin{cases} \frac{EA}{L}(l - L) - c(\dot{l} - \dot{L}_t) & \text{if } l > L \text{ and } T > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Integration Into a Multibody Simulation

When considering a multibody simulation, the pulley represented by cable node $\mathbf{P}_i$ can be attached to one of the simulated bodies. In that case, the dynamics of that body is affected by the tension of the cable acting as a force at $\mathbf{P}_i$. The force that is applied on the body is the result of the tension of the segments incident on that node:

$$\mathbf{F}_i = T\mathbf{d}_i - T\mathbf{d}_{i-1}, \quad \text{for } i = 1, \ldots, N-1; \quad (2)$$
$$\mathbf{F}_0 = T\mathbf{d}_0; \quad (3)$$
$$\mathbf{F}_N = -T\mathbf{d}_{N-1}. \quad (4)$$

The precise formulation used to apply an external force on a body at a given point depends on the particular modeling methodology that is used for the multibody system, and cannot be discussed here. See [Sha01] for an overview of the most common formulations in constrained multibody dynamics. The procedure to simulate the cable within a multibody system simulation can be depicted as follows:

1. Calculate cable length $l = \sum l_i$ and its change rate $\dot{l}$.
2. Calculate tension T using (1).
3. For $i = 1, \ldots, N-1$, if node $\mathbf{P}_i$ is attached to a body, then

   - compute $\mathbf{F}_i$ using (2–4);
   - apply $\mathbf{F}_i$ as an external force.

4. Perform the simulation step of the multibody system.

This scheme provides the same functionality as the one proposed in [SL07] and can be used as a new approach to massless cables. However, while the model presented in [SL07] is based on a particular multibody dynamics formulation, our model has the advantage that it can be easily integrated into complex simulations regardless the multibody modeling methodology chosen.

3.2. Oscillation of a Suspended Cable

A cable suspended by between points can be represented by a curve in space. Our model provides a physically based procedure to animate this curve in an interactive way. First, a two-dimensional horizontal model is considered. Then, its extension to the three-dimensional case with arbitrary orientation in space is explained.

Cable Oscillation in Two Dimensions

An elastic cable of linear density ρ and undeformed length $L > 0$ has its ends attached to points $P_0 = (x_0, z_0)$, and $P_1 = (x_1, z_1)$, with $x_1 = x_0 + l$, with $l > L$ so that the cable is under tension T. Without loss of generality, we shall consider that $x_0 = 0$. The external actions on the cable (e.g. external forces, or the movement of the cable ends) are assumed to act only vertically. Under this assumption, a cable point $s \in [0, l]$ can only move along z axis (see Figure 2) and its position can be expressed by its vertical coordinate $v(s, t)$. Let $a^2 = T/\rho$. The dynamics of the cable depicted above can be described by means of the wave equation [Fol92]:

$$v_{tt} + bv_t = a^2 v_{ss} + F(s, t);$$
$$v(0, t) = g(t); \quad v(l, t) = h(t), \tag{5}$$

where $F(s, t)$ is the vertical external force, $g(t)$ and $h(t)$ express the vertical position of the ends of the cable segment (see Figure 2) and bv_t is a damping term.
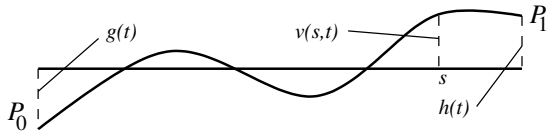

Figure 2: *Variables of the oscillation model.*

In order to ease implementation, equation (5) will be scaled so that l does not appear in the boundary conditions. In order to do so, the change of variable $xl = s$ is done, with $x \in [0, 1]$, leading to:

$$v_{tt} + bv_t = \frac{a^2}{l^2} v_{xx} + F(xl, t);$$
$$v(0, t) = g(t); \quad v(1, t) = h(t). \tag{6}$$

This model is actually a simplification; as a consequence of introducing a variable length $l = l(t)$ in the renormalization, $x = l(t)s$, additional lower order terms appear in equation (6). However, these terms can be safely neglected under the assumption of a high tension-to-density rate (high values of a^2) [GFPCMD07].

Although the wave equation (6) considers an almost horizontal cable, this model can be placed in an arbitrary position in space by projecting forces and displacements onto a local reference frame. This reference frame will be defined

so that points P_0 and P_1 are involved, allowing the frame to be updated properly as the points move. In order to do so, an additional transformation is introduced next. As a result, boundary conditions in (6) become homogeneous and the displacement of the cable ends is introduced as an additional term in the external forces.

Let us consider the segment that joins P_0 and P_1 in (6), given by $r(x, t) = g(t)(1 - x) + h(t)x$, and let $u(x, t) = v(x - t) - r(x, t)$; $\forall x \in [0, 1]$. Function $u(x, t)$ represents the vertical distance of the solution $v(x, t)$ to the segment and, hence, $u(0, t) = 0$; $u(1, t) = 0$ (see Figure 3). Taking the partial derivatives of u and introducing them in (6) the model can be expressed as

$$u_{tt} + bu_t = \frac{a^2}{l^2} u_{xx} + R(x, t);$$
$$u(0, t) = 0; \quad u(1, t) = 0; \tag{7}$$

where $x \in [0, 1]$ and

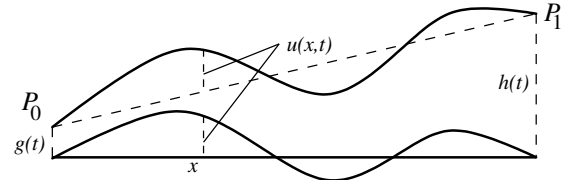$$R(x, t) = F(xl, t) - r_{tt}(x, t) + br_t(x, t)$$

Figure 3: *Variables of the transformed oscillation model.*

By means of these two transformations, and under the assumption of a cable under high tension, a general wave equation (5) has been transformed into a model in the interval $[0, 1]$ with homogeneous boundary conditions (7). In the rest of the paper this last model will be used.

Cable Oscillation in Three Dimensions

Next, the general case of a cable suspended in space is addressed. Let a cable of length L suspended from point $\mathbf{P}_0 \in \mathbf{R}^3$ to point $\mathbf{P}_1 \in \mathbf{R}^3$, and $l = ||\mathbf{P}_1 - \mathbf{P}_0|| > L$. Let $\mathbf{d} = (\mathbf{P}_1 - \mathbf{P}_0)/l$ be the unitary vector that points in the direction of the segment that links both points. Considering a pendulum as an a approximation to the movement of a crane payload, the main accelerations act in the plane defined by $\mathbf{d}$ and the gravity force. For this reason, the most noticeable oscillations of the cable take place within this plane, while lateral oscillations are less relevant (e.g. the swing of an horizontal cable during boom rotation). In order to exploit this property, the following local coordinate frame is defined:

$$\mathbf{e}_1 = \mathbf{d},$$
$$\mathbf{e}_2 = \frac{(0, 0, 1) - (0, 0, 1)\mathbf{e}_1^T \mathbf{e}_1}{||(0, 0, 1) - (0, 0, 1)\mathbf{e}_1^T \mathbf{e}_1||},$$
$$\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2.$$

Vector $\mathbf{e}_2$ is, by definition, perpendicular to $\mathbf{e}_1$ and it is contained within the plane defined by e_1 and the vertical axis. If $\mathbf{e}_2$ is undefined (when $\mathbf{e}_1 = (0,0,\pm1)$) it is taken so that it provides the maximum continuity to its evolution. According to this local frame, cable displacement is only allowed in the plane defined by vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Then, a point of the cable is uniquely determined by its distance from $\mathbf{P}_0$ along $\mathbf{e}_1$, denoted as x, and its distance to the segment $\overline{\mathbf{P}_0\mathbf{P}_1}$, denoted as $u(x)$ (Figure 4). Under this assumption, the planar oscillation model (7) can be used projecting the external forces and the movement of points $\mathbf{P}_0$ and $\mathbf{P}_1$ into this plane.
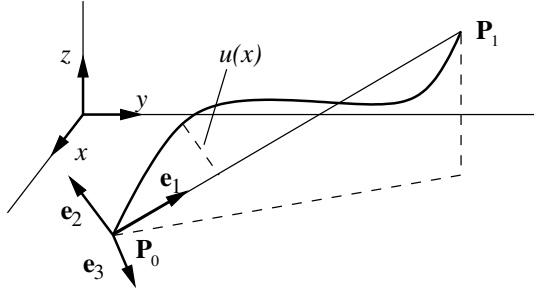

Figure 4: *The oscillation model in three dimensions is projected onto the plane defined by $\mathbf{e}_1$ and $\mathbf{e}_2$. A point of the cable can be uniquely be determined by its distance from $\mathbf{P}_0$ along the direction of $\mathbf{e}_1$ and its perpendicular distance to the segment $\overline{\mathbf{P}_0\mathbf{P}_1}$.*

3.3. Coupling of the Two Models

Recall the problem of a cable passing through a set of nodes $\mathbf{P}_i$, $i = 0,\ldots,N$. The piece of cable suspended between every pair of consecutive nodes, $\mathbf{P}_i$ and $\mathbf{P}_{i+1}$, is no longer considered as a straight line, but a curve in the plane $\langle\mathbf{e}_1,\mathbf{e}_2\rangle$ determined by cable oscillation. The evolution of this curve is defined by means of the oscillation model (7) described above, see Figure 5.

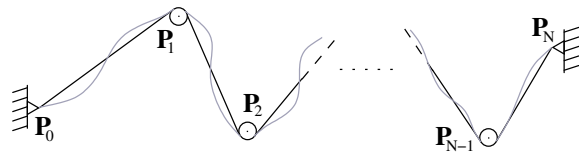**Figure 5:** *The oscillation model is superposed onto every segment of the* cable-and-pulleys *simulation scheme.*

Notice that now the actual cable length between two nodes is no longer their distance but the arc length of the solution $u(x,t)$, denoted as $\hat{l}_i$. The vector $\mathbf{e}_2$ associated to segment i will be denoted as $\mathbf{e}_2^i$. Therefore, the procedure to compute the cable simulation considering this model is:

1. Calculate cable length $l = \sum \hat{l}_i$ and its change rate.
2. Calculate tension T using (1).
3. For $i = 1,\ldots,N-1$,
   - if node $\mathbf{P}_i$ is attached to a body, then
     - compute $\mathbf{F}_i$ and apply it as an external force;
   - project the gravity and the variation of $\mathbf{P}_i$, $\forall i = 0,\ldots,N-1$ onto the associated $\mathbf{e}_2^i$;
   - using the computed value of T, R_i and l_i, evolve one step of the wave equation model (7);
   - update the value of the arc length $\hat{l}_i$.
4. Perform the simulation step of the multibody system.

This procedure completes the model that allows simulation of elevation cables in real-time. In the next section some remarks are given on the implementation and on its use on a real-time virtual environment.

4. Implementation Details

The integration of the partial differential equation (7) can be done by using different methodologies. In this section the numerical method used is explained, which achieves the goals for efficiency and stability. Also, details are given on collision detection and rendering.

4.1. Numerical Scheme and Stability

The variable coefficient wave equation (7) presented in Section 3 is integrated by means of a finite difference method. The solution is discretized as $u^m(t) = u(x^m,t)$, where $\{x^m = hm,\ m = 0,\ldots,M\}$, $h = \frac{1}{M}$, is a grid on the interval $[0,1]$. Time is also discretized with a fixed time step k. The proposed scheme avoids situations of instability while keeping both efficiency and robustness under input discontinuities.

For efficiency reasons, the explicit central-time-central-space difference scheme is used. This method is stable if and only if $\lambda = \frac{h}{k}$ meets the condition $\lambda a \leq 1$, where a is the coefficient of the wave equation (5) [Str04], p. 194. This stability condition implies that, given λ, the numerical scheme becomes instable only if the value of a increases, leading to high frequency oscillations of the cable. In such situations the oscillation amplitude will decay quickly due to the damping term b. Thus, in the situations that lead to numerical instability, the difference between the solution of (5) and a straight line will be hardly observable in a computer graphics application; hence the oscillation model is less relevant. Exploiting this property, stability is guaranteed as follows. During the simulation, the stability condition $\lambda\sqrt{T/\rho}/l < \sqrt{2}/2$ is checked at every time step. If it is not met, the solution of the wave equation is set to $u(x) = 0$ and the integration is suspended. As soon as the stability condition is held again, the integration is resumed, starting again from $u(x) = 0$.

4.2. Collision Detection

In order to get a highly realistic interactive simulation, it is necessary that cables collide against the rest of the scenery objects. Just for the shake of completeness, it is worth presenting a collision detection approach that is suitable to be used with the model proposed.

Collision detection is performed using two levels of resolution, allowing a more efficient implementation. The segment between node i and $i+1$ is enveloped by a bounding cylinder of length l_i and radius $r = \max(|u_i^m|)$. If a collision is detected, the cylinder is replaced by a new set of cylinders defined by the discretization of the cable u_i^m. Again, these geometries are checked for collision with the same object that collided the envelope. If one of these cylinders is in contact with the object, a new node is inserted in the cable, as if a pulley had been placed at the contact point.
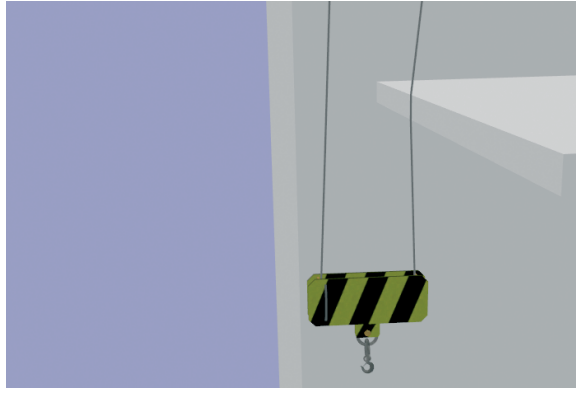

Figure 6: *A tower crane cable collision.*

The new pulley prevents the cable from penetrating the object, but also it must allow them to separate. At the end of every simulation step, if the cable and the object have started to move apart, the new pulley is removed. In order to detect it, when the pulley is inserted a *normal* vector $\mathbf{n}$ is computed and stored. This vector is perpendicular to the surface of the object colliding with the cable on the inserted node $\mathbf{P}_i$. The tension of the cable applied on $\mathbf{P}_i$ points to the direction $\mathbf{t} = \mathbf{d}_i - \mathbf{d}_{i-1}$. If $\mathbf{t} \cdot \mathbf{n} \geq 0$, the cable is moving away from the body.

4.3. Cable Rendering

A line is a natural primitive to represent a cable. The thickness of the line can be adjusted to fit the apparent size of the cable, according to its distance to the eye point. However, different parts of the cable may be at different distances from the observer, even in the same cable segment (e.g. a pendulum observed from above or the boom of a tower crane observed from the cabin), see Figure 7. Moreover, a line does not provide the required feeling of volume when the cable

is close to the viewer. For these reasons, it is better to represent a cable by means of cylinders; the shape of the cable is properly reproduced and textures and materials can be used. However, a considerable aliasing is observed when the apparent thickness of the cable is just of a few pixels; it appears as a dashed line which flickers as the cable moves.
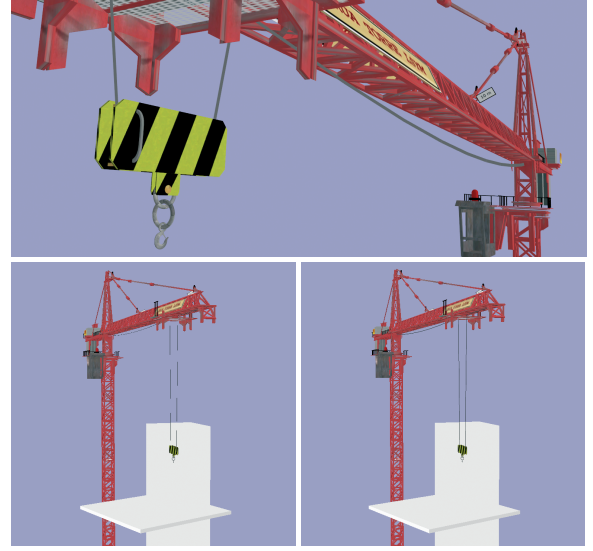

Figure 7: *The thickness of a line cannot be adjusted properly if the distance to the observer covers a wide range (above). The use of cylinders causes the cable to be seen as a dashed line if no hardware anti-aliasing is used (below, left). With our approach the inner line covers the gaps (below, right).*

In order to overcome these problems, we propose a hybrid approach: both the cylinders and the line primitive are rendered. With this method, when the distance to the cable is short the central line is hidden by the cylinders, but when the distance to the cable is long, the line fills the gaps that appear in the representation based only on cylinders. This approach provides a smooth transition between both methods. See Figure 7.

5. Evaluation of the Model

As it has been stated before, the main concern of this work is not to develop a very accurate model, but to obtain feasible simulations of cranes and other elevation machinery. Anyhow, the behavior of the model must be qualitatively correct and must reflect the main physical properties of the real system. In this section, some properties of the model are analyzed in order to check its correctness and the computational cost of the numerical scheme is studied to confirm its suitability for real-time simulations. The tests have been done using a tower crane elevation system model.

5.1. Energy Dissipation

The evolution of the total energy of the hook is measured in the test system and it is compared to the same value in the massless model. In both cases, the hook is placed at the lowest position at which tension is 0 and all the cable segments are set as straight lines. Then, the system is released under the effect of gravity. The vertical position of the hook oscillates until an equilibrium between tension and hook weight is reached. Figure 8 shows the evolution of the energy of the hook for different values of cable density. It is worth noting that, in the cable with mass, tension has also to compensate cable weight in the horizontal segments, causing different modes of oscillation for different densities at the beginning of the experiment. Anyhow, it can be observed that in the long term the decrease rate of the total energy for the massless model and for the different densities are the same.
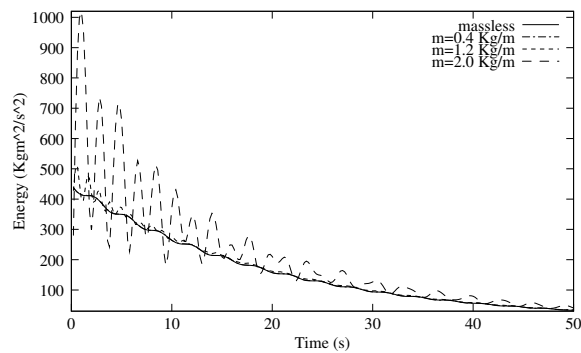

Figure 8: *Evolution of the total energy of the hook.*

The pendulum frequency has also been tested for different values of the cable density and of the damping parameters. No indication that the oscillation of the cable should influence on this value is found in the bibliography and our tests confirm this hypothesis.

5.2. Effect of Mass in Cable Dynamics

The introduction of cable mass has interesting effects on lift systems that cannot be reproduced using massless cables. As the energy decay test has shown, different amplitude and frequency oscillations can be observed in the energy of the payload for different cable densities. Furthermore, other effects are also observable, as shown next.

Consider the tower crane in Figure 9; if the hook lays on the floor, the catenary in the two horizontal cable segments is large. When the lift movement starts, a short period of time elapses until the hook begins to elevate. This effect is produced because the reduction of cable length has to reduce the catenary before tension compensates the weight of the hook. In our model, the introduction of mass makes this effect noticeable, which is not reproduced by any other model. This
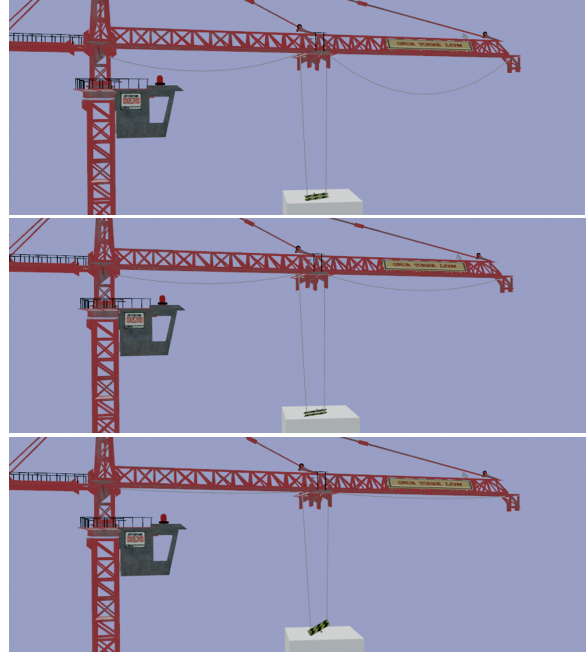

Figure 9: *The effect of cable mass can be observed in the sequence. The hook lays on the platform until all the catenary is reduced.*

effect has considerable influence on the quality of a crane model when it is used for training purposes, as it determines response times and the vertical oscillation of the payload.

5.3. Computational Complexity

The computational complexity of the model is now discussed. For every segment, the cost of the finite difference scheme is linear respect to the number of spatial subdivisions, M, and the computation of its arc length involves a number of calculations that are also linear with M. Once the arc length has been computed, only a constant number of computations are necessary to apply forces at every node. Thus, our model is linear with the sum of total subdivisions used to represent the cable oscillation, $N \times M$, which is the number of bodies that should be used to obtain an equivalent resolution in a multibody representation. Although some multibody formulations achieve linear cost respect to the number of bodies [Had06, SL08], these methodologies are not that efficient when closed loops are formed, e.g. when two pulleys are attached to the same body. Indeed, to our knowledge, the use of pulleys has not been developed for strands or cable models using multibody dynamics.

The model has been tested for performance by running 100000 simulation steps for different cable sizes. The number of nodes, N, and of interval subdivisions, M, have been varied in a range from 10 to 100. The computational cost

of the simplest model is below 0.02ms per time step, while the heaviest model, with 100 nodes and 100 subdivisions per node, spends 1.3ms per time step. This model would be equivalent to a multibody model with 10000 bodies and links. As a reference, the multibody model by Servin and Lacoursière [SL08], which scales linearly with the number of nodes, has a computational cost of around 1ms for a cable with 24 segments.

The cable model has also been integrated into a production tower crane simulator with no noticeable performance reduction, while the quality of the simulation has increased and the training capabilities of the application have been extended as a result of the new cable features (see Figure 10).

Figure 10: *A screen shot of the tower crane simulator developed at University of Valencia.*

5.4. Limitations of the model

The model focuses on a feasible real-time animation of the oscillations of a cable when it is under high tension, such as crane elevation cable. In order to gain efficiency and stability, some assumptions are made, that are acceptable in this situation. Bending and torsion of the cable are not considered. Also, oscillation in the horizontal direction is discarded, considering only movement within a vertical plane, as it is the most noticeable one. Instability is also prevented by assuming that oscillation is less noticeable in the situations when it can arise. The experience in our application is positive, but it is clear that this method can be less adequate in other situations.

6. Conclusions

We have presented a two-layered model for crane cables that separates the computation of cable tension and cable oscillation. Using this approach instability issues have been

avoided, allowing simulation in real-time. Limitations of previous massless cables have been overcome, reproducing some important behavior of cranes which cannot be predicted with any previous model. The dynamics of the model has been discussed and validated and its performance has been analyzed and supported with numerical experiments. Also, the integration of the model into a complex simulation environment has been addressed, including details on collision detection and rendering.

In future research, we intend to extend the model to consider another features, such as cable swing and sway or pulley friction. Another improvement of the model would be a model for the simulation of very low tension situations; the oscillation model would be switched when tension falls below a given threshold.

Acknowledgments

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